DaTALab NorthCOVID-19 Simulation

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https://covid.datalab.science/

Urban Equations

For the *Susceptible* population (S), the initial value is equal to $TotalPopulation - InitialInfected$ ($N - N_i$). The change in the ratio of susceptible individuals s to the total population N (i.e., S/N) over time (measured in days) is equal to the following:

$$\frac{d(s)}{dt} = -I_f \tag{1}$$

Where I_f is only used if the total number of people that have been infected $TotalInfected$ (N_t) is less than a ratio of the N referred to as $MaxInfectedRatio$ (α_1). The formula for I_f is the following:

$$I_f = \frac{\bar{c} \times \tau \times I \times S}{N} \tag{2}$$

Where I is the number of $Infectious$ individuals, $\bar{c}$ is the $ContactRate$ of individuals 1 person will contact in a day, and τ is the $Infectivity$ percent chance a $Susceptible$ individual will contract the infection once they've been in contact with an infected individual. The initial value for I is equal to N_i and the change in the ratio i (i.e., I/N) over time is defined as follows:

$$\frac{d(i)}{dt} = I_f - H_f - R_f \tag{3}$$

Where H_f is the $HospitalizationFlow$ of infected individuals that require hospitalization and is defined as follows:

$$H_f = I_f \times \alpha_2 \tag{4}$$

Where α_2 is the $HospitalizationRate$ percent of infected individuals that will need to go to the hospital. As for R_f, that is the $RecoveredFlow$ of individuals who recovered from the infection and is defined as follows:

$$R_f = i - H_f \tag{5}$$

This leads to an *intermediate* state where the individual stays for ν days ($AverageIllnessDuration$) before finally going to the *Recovered* state. This is to simulate the number of days that an infected individual will need to recover from the infection as they are still counted as being in the $Infectious$ population for this time period.

The number of $InfectedHospitalized$ individuals that have been sent to the hospital is referred to as i_h and the change over time for this population is defined as follows:

$$\frac{d(i_h)}{dt} = H_f - I_{wf} - I_{cf} - H_{xf} \tag{6}$$

Where I_{wf} is the $InfectedWardFlow$ of hospitalized individuals that need to go to the ward and is defined as follows:

$$I_{wf} = i_h \times \alpha_3 \tag{7}$$

Where α_3 is the $WardRate$ percent of hospitalized individuals that will need to go to the ward. I_{cf} is the $InfectedIcuFlow$ of hospitalized individuals that need to go to the ICU and is defined as follows:

$$I_{cf} = i_h \times \alpha_4 \tag{8}$$

Where α_4 is the $IcuRate$ percent of hospitalized individuals that will need to go to the ICU. H_{xf} is the $HospitalizedDeathFlow$ of hospitalized individuals who will die from the infection defined as follows:

$$H_{xf} = i_h \times \alpha_{12} \tag{9}$$

Where α_{12} is the $HospitalizedDeathRate$ percent of hospitalized individuals who will die from the infection. Note that the summation of α_{12}, α_4 ($IcuRate$), and α_3 ($WardRate$) **cannot** exceed 1.00. This flow leads to an *intermediate* state where the individuals stay for ν_x days ($AverageHospitalizationDeathDuration$) before moving to the *Death* state.

The $InfectedWard$ number of individuals in the ward is referred to as i_w

and the change over time for this population is defined as follows:

$$\frac{d(i_w)}{dt} = I_{cro} + I_{cwf} + I_{wf} - I_{rf} - I_{wcf} \tag{10}$$

Where I_{cwf} is the $InfectedIcuWardFlow$ of individuals moving from the ICU to the ward and is defined as follows:

$$I_{cwf} = i_c + I_{cf} - I_{xf} \tag{11}$$

This leads to an *intermediate* state where the individual stays for α_5 days ($IcuStayRate$) before finally going to the $InfectedWard$ state. This is to simulate the surviving individuals' stay in the ICU as they are still counted as being in the $InfectedIcu$ population for this time period. Once reaching the $InfectedWard$ state, the individuals are, again, kept in an intermediate state where, over the course of α_{11} days ($DischargeStayDuration$), they are finally moved to the *Recovered* state. This is to simulate the post-discharged recovery duration for an ICU patient as they are still counted as being in the $InfectedWard$ population for this time period. I_{xf} is the $DeathFlow$ of individuals moving to the death state and is defined as follows:

$$I_{xf} = (i_c + I_{cf}) \times \alpha_6 \tag{12}$$

Where α_6 is the $DeathRate$ percent chance that an individual in the ICU will die, and, lastly, I_{wcf} is the $InfectedWardIcuFlow$ of individuals moving from the ward to the ICU and is defined as follows:

$$I_{wcf} = i_w \times \alpha_9 \tag{13}$$

Where α_9 is the $WardToIcuRate$ percent of individuals that will need to move from the ward state to the ICU state.

Next, I_{rf} is the $InfectedRecoveredFlow$ of individuals moving from the ward to the recovered state and is defined as follows:

$$I_{rf} = i_w - I_{wcf} \tag{14}$$

This leads to an *intermediate* state where the individual stays for α_{10} days ($HospitalStayDuration$) before finally going to the *Recovered* state. This is to simulate the ward individuals' stay as they are still counted as being in the $InfectedWard$ population for this time period.

The $InfectedIcu$ individuals in the ICU is referred to as i_c and the change over time for this population is defined as follows:

$$\frac{d(i_c)}{dt} = I_{cf} + rI_{cf} + I_{wcf} + rI_{wcf} - I_{cwf} - rI_{cwf} - I_{xf} - rI_{xf} - I_{cro} - I_{cxo} \quad (15)$$

Where I_{cro} is only used when the i_c population reaches its max capacity $MaxIcuCapacity$ (N_c) where $InfectedIcuRecoveryOverflow$ number of individuals is overflowed from the ICU to the ward state and is defined as follows:

$$I_{cro} = i_c + I_{cf} + I_{wcf} - I_{xf} - I_{cwf} - N_c - I_{cxo} \quad (16)$$

This leads to an *intermediate* state where the individual stays for α_8 days ($IcuOverflowRecoveryOffsetDuration$) before finally going to the $InfectedWard$ state. This is to simulate the individual taking α_8 additional days on top of α_{10} to recover as they are still counted as being in the $InfectedWard$ population for this time period. I_{cxo} is also only used when i_c exceeds N_c to overflow $InfectedIcuDeathOverflow$ number of individuals to the death state and is defined as follows:

$$I_{cxo} = (i_c + I_{cf} + I_{wcf} - I_{xf} - I_{cwf} - N_c) \times (1 - \alpha_7) \quad (17)$$

Where α_7 is the $IcuOverflowRecoveryRate$ percent chance that an individual, that needed to go to the ICU but would've exceeded the N_c, will survive.

Lastly, rI_{cf}, rI_{wcf}, rI_{cwf}, and rI_{xf} are used for incoming ($rI_{cf} + rI_{wcf}$) and outgoing ($-rI_{wcf} - rI_{xf}$) **rural** patients in the **urban** ICU. By default, the incoming rural flows are directed to the **urban** ICU. If the **urban** ICU would become full with the incoming flow of **rural** patients, these flows are redirected to the **rural** ICU. Within this logic, the model does try to transfer a portion of the **rural** patients such that the **urban** ICU would not overflow if it's possible at all. This takes into account all of the other incoming flows to the urban ICU when deciding this. For the outgoing flows, they are always applicable to both the **rural** ICU and the **rural** patients in the **urban** ICU. Since this population ultimately ends up back in the **rural** portion of our model, they do not need to be considered when summing the differentials for the **urban** portion. The definition for these variables are defined in Equations 33, 38, 36, and 37 respectively.

The ratio of the total population of *Recovered* (R) individuals from the infection to the total population is referred to as r (i.e., R/N)and the change over time for this population is defined as follows:

$$\frac{d(r)}{dt} = R_f + I_{rf} \tag{18}$$

The population of $InfectedDeath$ individuals that have died from the infection is referred to as x and the change over time for this population is defined as follows:

$$\frac{d(x)}{dt} = I_{xf} + I_{cxo} + H_{xf} \tag{19}$$

Finally, let's check that the differential equations add to zero- we will do two at a time to follow it easily and make variables that cancel out red:

$$\frac{d(s)}{dt} + \frac{d(i)}{dt} = -I_f + I_f - H_f - R_f \tag{20}$$

$$-H_f - R_f + \frac{d(i_h)}{dt} = -H_f - R_f + H_f - I_{wf} - I_{cf} - H_{xf} \tag{21}$$

$$-R_f - I_{wf} - I_{cf} + \frac{d(i_w)}{dt} = -R_f - I_{wf} - I_{cf} - H_{xf} + I_{cro} + I_{cwf} + I_{wf} - I_{rf} - I_{wcf} \tag{22}$$

$$\begin{aligned} -R_f - I_{cf} + I_{cro} + I_{cwf} - I_{rf} - I_{wcf} + \frac{d(i_c)}{dt} = -R_f - I_{cf} - H_{xf} + I_{cro} + \\ I_{cwf} - I_{rf} - I_{wcf} + I_{cf} + I_{wcf} - I_{cwf} - I_{xf} - I_{cro} - I_{cxo} \end{aligned} \tag{23}$$

$$-R_f - I_{rf} - I_{xf} - I_{cxo} + \frac{d(r)}{dt} = -R_f - H_{xf} - I_{rf} - I_{df} - I_{cxo} + R_f + I_{rf} \tag{24}$$

$$-I_{df} - I_{cxo} + \frac{d(x)}{dt} = -H_{xf} - I_{xf} - I_{cxo} + I_{xf} + I_{cxo} + H_{xf} = 0 \tag{25}$$

Rural Equations

Although identical to the **urban** equations, the **rural** equations have their own, independent parameters that make defining them a necessity for completeness. The $RuralSusceptible$ population (rS), the initial value is equal to $RuralTotalPopulation - RuralInitialInfected$ ($rN - rN_i$). The change in the ratio of susceptible individuals rs to the total population rN (i.e., rS/rN) over time (measured in days) is equal to the following:

$$\frac{d(rs)}{dt} = -rI_f \tag{26}$$

Where rI_f is only used if the total number of people that have been infected $RuralTotalInfected$ (rN_t) is less than a ratio of the rN referred to as $RuralMaxInfectedRatio$ ($r\alpha_1$). The formula for rI_f is the following:

$$rI_f = \frac{\overline{rc} \times r\tau \times rI \times rS}{rN} \tag{27}$$

Where rI is the number of $RuralInfectious$ individuals, $\overline{rc}$ is the $RuralContactRate$ of individuals 1 person will contact in a day, and $r\tau$ is the $RuralInfectivity$ percent chance a $RuralSusceptible$ individual will contract the infection once they've been in contact with an infected individual. The initial value for rI is equal to rN_i and the change in the ratio ri (i.e., rI/rN) over time is defined as follows:

$$\frac{d(ri)}{dt} = rI_f - rH_f - rR_f \tag{28}$$

Where rH_f is the $RuralHospitalizationFlow$ of infected individuals that require hospitalization and is defined as follows:

$$rH_f = rI_f \times r\alpha_2 \tag{29}$$

Where $r\alpha_2$ is the $RuralHospitalizationRate$ percent of infected individuals that will need to go to the hospital. As for rR_f, that is the $RuralRecoveredFlow$ of individuals who recovered from the infection and is defined as follows:

$$rR_f = ri - rH_f \tag{30}$$

This leads to an *intermediate* state where the individual stays for $r\nu$ days ($RuralAverageIllnessDuration$) before finally going to the $RuralRecovered$

state. This is to simulate the number of days that an infected individual will need to recover from the infection as they are still counted as being in the $RuralInfectious$ population for this time period.

The number of $RuralInfectedHospitalized$ individuals that have been sent to the hospital is referred to as ri_h and the change over time for this population is defined as follows:

$$\frac{d(ri_h)}{dt} = rH_f - rI_{wf} - rI_{cf} - rH_{xf} \tag{31}$$

Where rI_{wf} is the $RuralInfectedWardFlow$ of hospitalized individuals that need to go to the ward and is defined as follows:

$$rI_{wf} = ri_h \times r\alpha_3 \tag{32}$$

Where $r\alpha_3$ is the $RuralWardRate$ percent of hospitalized individuals that will need to go to the ward. rI_{cf} is the $RuralInfectedIcuFlow$ of hospitalized individuals that need to go to the ICU and is defined as follows:

$$rI_{cf} = ri_h \times r\alpha_4 \tag{33}$$

Where $r\alpha_4$ is the $RuralIcuRate$ percent of hospitalized individuals that will need to go to the ICU. rH_{xf} is the $RuralHospitalizedDeathFlow$ of hospitalized individuals who will die from the infection defined as follows:

$$rH_{xf} = ri_h \times r\alpha_{12} \tag{34}$$

Where $r\alpha_{12}$ is the $RuralHospitalizedDeathRate$ percent of hospitalized individuals who will die from the infection. Note that the summation of $r\alpha_{12}$, $r\alpha_4$ ($RuralIcuRate$), and $r\alpha_3$ ($RuralWardRate$) **cannot** exceed 1.00. This flow leads to an *intermediate* state where the individuals stay for $r\nu_x$ days ($RuralAverageHospitalizationDeathDuration$) before moving to the $RuralDeath$ state.

The $RuralInfectedWard$ number of individuals in the ward is referred to as ri_w and the change over time for this population is defined as follows:

$$\frac{d(ri_w)}{dt} = rI_{cro} + rI_{cwf} + rI_{wf} - rI_{rf} - rI_{wcf} \tag{35}$$

Where rI_{cwf} is the $RuralInfectedIcuWardFlow$ of individuals moving from the ICU to the ward and is defined as follows:

$$rI_{cwf} = ri_c + rI_{cf} - rI_{xf} \tag{36}$$

This leads to an *intermediate* state where the individual stays for $r\alpha_5$ days ($RuralIcuStayRate$) before finally going to the $RuralInfectedWard$ state. This is to simulate the surviving individuals' stay in the ICU as they are still counted as being in the $RuralInfectedIcu$ population for this time period. Once reaching the $RuralInfectedWard$ state, the individuals are, again, kept in an intermediate state where, over the course of $r\alpha_{11}$ days ($RuralDischargeStayDuration$), they are finally moved to the $RuralRecovered$ state. This is to simulate the post-discharged recovery duration for an ICU patient as they are still counted as being in the $RuralInfectedWard$ population for this time period. rI_{xf} is the $RuralDeathFlow$ of individuals moving to the death state and is defined as follows:

$$rI_{xf} = (ri_c + rI_{cf}) \times r\alpha_6 \tag{37}$$

Where $r\alpha_6$ is the $DeathRate$ percent chance that an individual in the ICU will die, and, lastly, rI_{wcf} is the $InfectedWardIcuFlow$ of individuals moving from the ward to the ICU and is defined as follows:

$$rI_{wcf} = ri_w \times r\alpha_9 \tag{38}$$

Where $r\alpha_9$ is the $RuralWardToIcuRate$ percent of individuals that will need to move from the ward state to the ICU state.

Next, rI_{rf} is the $RuralInfectedRecoveredFlow$ of individuals moving from the ward to the recovered state and is defined as follows:

$$rI_{rf} = ri_w - rI_{wcf} \tag{39}$$

This leads to an *intermediate* state where the individual stays for $r\alpha_{10}$ days ($RuralHospitalStayDuration$) before finally going to the $RuralRecovered$ state. This is to simulate the ward individuals' stay as they are still counted as being in the $RuralInfectedWard$ population for this time period.

The $InfectedIcu$ individuals in the ICU is referred to as ri_c and the change over time for this population is defined as follows:

$$\frac{d(ri_c)}{dt} = rI_{cf} + rI_{wcf} + rI_{wcf} - rI_{cwf} - rI_{xf} - rI_{cro} - rI_{cxo} \tag{40}$$

Where rI_{cro} is only used when the ri_c population reaches its max capacity $RuralMaxIcuCapacity$ (rN_c) where $RuralInfectedIcuRecoveryOverflow$ number of individuals is overflowed from the ICU to the ward state and is defined as follows:

$$rI_{cro} = ri_c + rI_{cf} + rI_{wcf} - rI_{xf} - rI_{cwf} - rN_c - rI_{cxo} \tag{41}$$

This leads to an *intermediate* state where the individual stays for $r\alpha_8$ days ($RuralIcuOverflowRecoveryOffsetDuration$) before finally going to the $RuralInfectedWard$ state. This is to simulate the individual taking $r\alpha_8$ additional days on top of ν to recover as they are still counted as being in the $RuralInfectedWard$ population for this time period. rI_{cxo} is also only used when ri_c exceeds rN_c to overflow $RuralInfectedIcuDeathOverflow$ number of individuals to the death state and is defined as follows:

$$rI_{cxo} = (ri_c + rI_{cf} + rI_{wcf} - rI_{xf} - rI_{cwf} - rN_c) \times (1 - r\alpha_7) \tag{42}$$

Where $r\alpha_7$ is the $RuralIcuOverflowRecoveryRate$ percent chance that an individual, that needed to go to the ICU but would've exceeded the rN_c, will survive.

The ratio of the total population of $RuralRecovered$ (rR) individuals from the infection to the total population is referred to as rr (i.e., rR/rN)and the change over time for this population is defined as follows:

$$\frac{d(rr)}{dt} = rR_f + rI_{rf} \tag{43}$$

The population of $RuralInfectedDeath$ individuals that have died from the infection is referred to as rx and the change over time for this population is defined as follows:

$$\frac{d(rx)}{dt} = rI_{xf} + rI_{cxo} + rH_{xf} \tag{44}$$